Coronal mass ejections (CMEs) are huge eruptions of solar plasma and magnetic field, often associated with active regions and solar flares. They propagate out through the corona, into the solar wind and can reach near-Earth space within a few days, where they are responsible for the largest solar wind speeds and magnetic field strengths observed. The magnetic field configuration of CMEs enables them to effectively couple with the terrestrial magnetosphere and consequently drive the largest geomagnetic storms. We look at the commonalities and differences between the extreme geomagnetic storms, and the CMEs which caused them, observed and modelled over the last solar cycle. Geoeffective CMEs generally have speeds far greater than the ambient solar wind and so drive interplanetary shock waves, which in turn accelerate solar energetic particles (SEPs). While this constitutes a further space weather threat, it also allows us to use observational proxies for SEPs to provide information on CME activity over the last century and thus better understand the statistics of extreme space weather events.