Vortex-Rossby waves (VRWs) and inertial gravity waves (IGWs) have been proposed to explain the propagation of spiral rainbands and the development of dynamical instability in tropical cyclones (TCs). In this study, a theory for mixed vortex-Rossby-inertial-gravity waves (VRIGWs) coexisting with VRWs and IGWs is developed by including both rotational and divergent flows in a shallow-water equations model. A cloud-resolving TC simulation is used to help simplify the radial structure equation for linearized perturbations, and then transform it to a Bessel equation with constant coefficients. A cubic frequency equation describing the three groups of allowable (radially discrete) waves is eventually obtained.

It is shown that low-frequency VRWs and high-frequency IGWs may coexist, but with separable dispersion characteristics, in the eye and outer regions of TCs, whereas mixed VRIGWs with inseparable dispersion and wave instability properties tend to occur in the eyewall. The mixed-wave instability, with shorter waves growing faster than longer waves, appears to explain the generation of polygonal eyewalls and multiple vortices with intense rotation and divergence in TCs. Results show that high-frequency IGWs would propagate at half their typical speeds in the inner regions with more radial “standing” structures. Moreover, all the propagating waves appear in the forms of spiral bands with different intensities as their radial widths shrink in time, suggesting that some spiral rainbands in TCs may result from the radial differential displacements of azimuthally propagating perturbations.