We analyse INTERMAGNET data about strongest geomagnetic storms observed during 23-rd solar cycle. We compare this vast data set both statistically and individually with solar, heliospheric and magnetospheric conditions compiled in our data base APEV (http://dbserv.sinp.msu.ru/apev/fullist.htm) with the aim to investigate general and specific properties of all events, which demonstrate common features and broad diversity of situations. Methods of analysis and dynamical visualization of big data sets based on fuzzy logic geo-informational approach, named “Discrete mathematical analysis” (DMA), will be presented. We bring new examples of the applications of geomagnetic data in solar-terrestrial physics and extend the discussion of existing limitations in this regard. Based on our analysis we conclude that geomagnetic proxies could serve as an important source of indirect information about solar and heliospheric activity in the past, when direct observations were not available. Necessity of the original (not corrected) data archives preservation and reanalysis for this purpose are especially stressed and illustrated in several instances related to the long-term behavior as well as extremely perturbed and quiet conditions. Reliability and accuracy of physical interpretations and models essentially depends on the quality of geomagnetic input information and assumed conditions on the ground, in the ionosphere, the magnetosphere, the solar wind and on the Sun. Well calibrated INTERMAGNET data in combination with other ground based measurements and multipoint satellite missions are promising for the future progress towards better and broader scientific and technical use of geomagnetic information.