The Darwin - de Sitter second-order theory, comparing to the Clairaut first-order theory, is considered as standard theory of equilibrium figures of the Earth interior, though there are also some other higher-order or more general development such as Denis (1989). However, all these theories can not deal with the contribution from the anti-axial-symmetric mass distribution, i.e. the spherical terms of non-zero order and odd degree.

The fact is that the inhomogenous and anti-symmetry of the outermost crust, topography and oceanic layers are too big to ignore. We developed here a new integrated formula to obtain the equilibrium figures to integrated third-order accuracy. In this formula, both the direct and indirect contribution of the anti-symmetric crust layer are included, as result, all the odd degree and non-zero order terms are included in the spherical harmonic expansion.

Using our new potential theory and real surface layers data, we re-calculate the geometrical flattening profile of the Earth interior and, as a by-product, the global dynamic flattening (H), which is an important quantity in research of rotating Earth and is related with precession constant, is obtained (1/306.68). The significant difference (previously 1.1%) between the value of H from observation and that from traditional theory is reduced, by about 2/3, to 0.38%.

Furthermore, this new theory is applied to derive the non-spheroidal figure of the core-mantle boundary and then its contribution to the free core nutation (FCN) is discussed.