According to the well-known conventions the vertical datum is defined as the equipotential surface for which the Earth gravity potential is constant. The aim of this contribution is to focus on differential geometry properties of equipotential surfaces and their relation to problems of physical nature in vertical datum definition. Within this concept one can apply a number of tools. The discussion mainly rests on the use of tensor calculus and also exterior differential forms. In particular the importance of Weingarten’s theorem in the theory of surfaces will be emphasized together with its essential tie to Brun’s equation (for gravity gradient), which is well known in physical geodesy. Also the role of Christoffel’s theorem will be mentioned. These considerations are of constructive nature and numerically their content will be demonstrated through the use of high performance and accuracy computations for gravity field models represented in terms of high degree and order expansions into series of ellipsoidal harmonics. The results will be interpreted globally and also in merging segments expressing regional and local features of the gravity field of the Earth. They may contribute to knowledge important for the realization of a World Height System.