Surface geomagnetic observation can determine up to degree $L = 14$ time-varying spherical harmonic coefficients of the poloidal magnetic field. Assimilation of these coefficients to numerical dynamo simulation could help us understand better the dynamical processes in the Earth's outer core, and to provide more accurate forecast of geomagnetic secular variations (SV). In our previous assimilation studies, only the poloidal magnetic field in the core is corrected by the observations in the analysis. Unobservable core state variables (the toroidal magnetic field and the core velocity field) are corrected via the dynamical equations of the geodynamo. Our assimilation experiments show that the assimilated core state converges near the CMB, implying that the dynamo state is strongly constrained by surface geomagnetic observations, and is pulled closer to the truth by the data.

We are now carrying out an ensemble of assimilation runs with 1000 years of geomagnetic and archeo/paleo magnetic record. In these runs the cross correlation between the toroidal and the poloidal magnetic fields is incorporated into the analysis. This correlation is derived from the physical boundary conditions of the toroidal field at the core-mantle boundary (CMB). The assimilation results are then compared with those of the ensemble runs without the cross-correlation, aiming at understanding two fundamental issues: the effect of the cross-correlation on (1) the convergence of the core state, and (2) the SV prediction accuracies. The constrained dynamo solutions will provide valuable insights on interpreting the observed SV, e.g. the near-equator magnetic flux patches, the core-mantle interactions, and possibly other geodynamic observables.