Certain algebraic combinations of single-scattering albedo and solar radiation reflected from, or transmitted through, vegetation canopies do not vary with wavelength. These are called "spectrally-invariant relationships" and are the consequence of wavelength independence of the extinction coefficient and scattering phase function in vegetation. This wavelength-independence does not in general hold in the atmosphere, but in cloud-dominated atmospheres the total extinction and total scattering phase function are only weakly sensitive to wavelength. We identify the atmospheric conditions under which the spectrally-invariant approximation can accurately describe the extinction and scattering properties of cloudy atmospheres. Validity of the assumptions and accuracy of the approximation are tested with 1D radiative transfer calculations using publicly available radiative transfer models: DISORT and SBDART. We show that, for cloudy atmospheres with cloud optical depth above 3, and for spectral intervals that exclude strong water vapor absorption, the spectrally-invariant relationships found in vegetation-canopy radiative transfer are valid to better than 5%. We discuss the physics behind this phenomenon, its mathematical basis, and possible applications to remote sensing and climate. This is the first step towards theoretical justification of a spectrally-invariant relationship recently found in ARM shortwave spectrometer observations and confirmed by SBDART simulations.